

## Reconciliation as a Tool for Decision Making within Decision Tree Related to Insolvency Problems

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### Abstract

**Purpose of the article:** The paper draws on the results of previous studies recoverability of creditor's claims, where it was research from debtor's point of view and his/her debts on the Czech Republic financial market. The company, which fell into a bankruptcy hearing, has several legislatively supported options how to deal with this situation and repay creditors money. Each of the options has been specified as a variant of a decision-making tree. This paper is focused on third option of evaluation – The reconciliation. The heuristic generates all missing information items. The result is then focused on the comparison and evaluation of the best ways to repay the debt, also including solution for the future continuation of the company currently in liquidation and quantification of percentage refund of creditors claim. A realistic case study is presented in full details. Further introduction of decision making with uncertainties in insolvency proceedings.

**Methodology/methods:** Solving within decision tree with partially ignorance of probability using reconciliation.

**Scientific aim:** Comparison and evaluation of the best ways to repay the debt, also including solution for the future continuation of the company currently in liquidation and quantification of percentage refund of creditors claim.

**Findings:** Predictions of future actions in dealing with insolvency act and bankruptcy hearing, quicker and more effective agreeing on compromises among all creditors and debtor.

**Conclusions:** Finding a best way and solution of repayment and avoiding of termination for both of interested parties (creditor and debtor).

**Keywords:** insolvency, restructuring, decision – making tree, probabilities, creditors claim, reconciliation

**JEL Classification:** G33, G34

## Introduction

In a previous paper (Poláček, 2015) revealed the percentage probability of recovery of debtors' debts back to creditors using decision-making tools – solutions with known probabilities and solutions with the use of the calculations of so. water probabilities. To verify these calculations was created a model situation in unknown or partially known probabilities – e.g. fuzzy reconciliation.

To calculate the amount of recoveries receivable were used three methods of calculation – Solving by the known probabilities (Poláček, 2015), then the use of so-called water probability (Poláček, 2015) and reconciliation. Each of the above methods of calculation issued several sub-results. That will later be given to all proportion and it will be determined the best method of balancing the claims against the creditor.

Each scenario we created by a decision tree is terminated by coagulating the percentage of the total amount of debt, depending on the selected criteria solutions. Data used to calculate the percentage degradation was obtained from statistical files and is the recovery of the amount of the total claim.

### 1. Decision – making tree

A decision tree is a method that can be used to make advance unpredictable choices, especially decisions that involve high costs and risks (Olivas, 2007).

Decision tree models include such concepts as nodes, branches, endpoints, strategy, payoff distribution, certain equivalent, and the rollback method.

**Nodes** are divided into root nodes, decision nodes and chance nodes (Magee, 1964). Root node, or also the first decision node, which is the beginning

of decision tree and so represents a “first” decision, in the used decision tree the root node representing the entering into insolvency. Decision – making nodes represent a firm decision and they are plotted as the squares compare to **chance nodes** are plotted as small circles; they represent an event that can result in two or more outcomes. Each outcome from chance node has its own probability, the total of all outcomes for a given chance node must equal 100% (or 1.0). Lines that connect nodes are called **branches**. A branch that goes from a decision node (and towards the right) is called decision branches. Similarly, branches that outcome from a chance node (and towards the right) are called chance branches. Each branch can represent different kind of strategy of pre – selected decision. The branch can lead to any of the three node types: decision node, chance node, or endpoint. The **Endpoints**, also known as terminate points are plotted as triangles and represent the termination of the decision tree with exact result, that can be represent as payoff values. For example in case study is it shown as future possibility of refund of creditors' claims (in case study percentages stands for payoff values)

### 2. Types of Insolvency Proceedings

Upon the declaration by the Insolvency Court of the debtor's insolvency, the insolvency is dealt with under one of the following types of the insolvency proceedings:

- Bankruptcy;
- Reorganisation; or
- Debt clearance.

In cases of bankruptcy, the debtor's assets are sold and the creditors' claims are proportionally satisfied using the output of the sale of assets. Unsatisfied

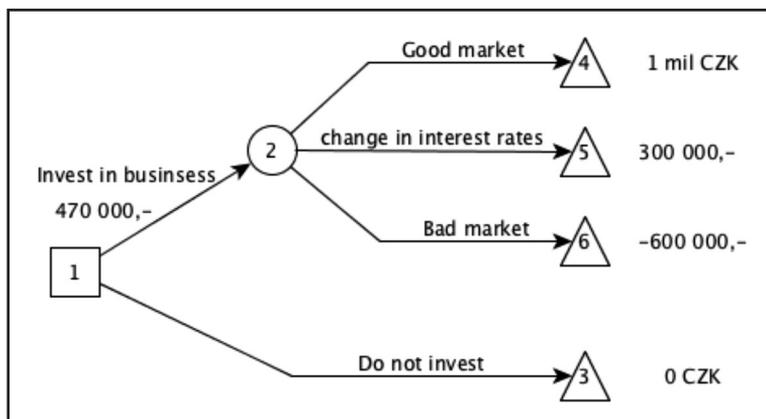


Figure 1. Decision tree. Source: Authors own work.

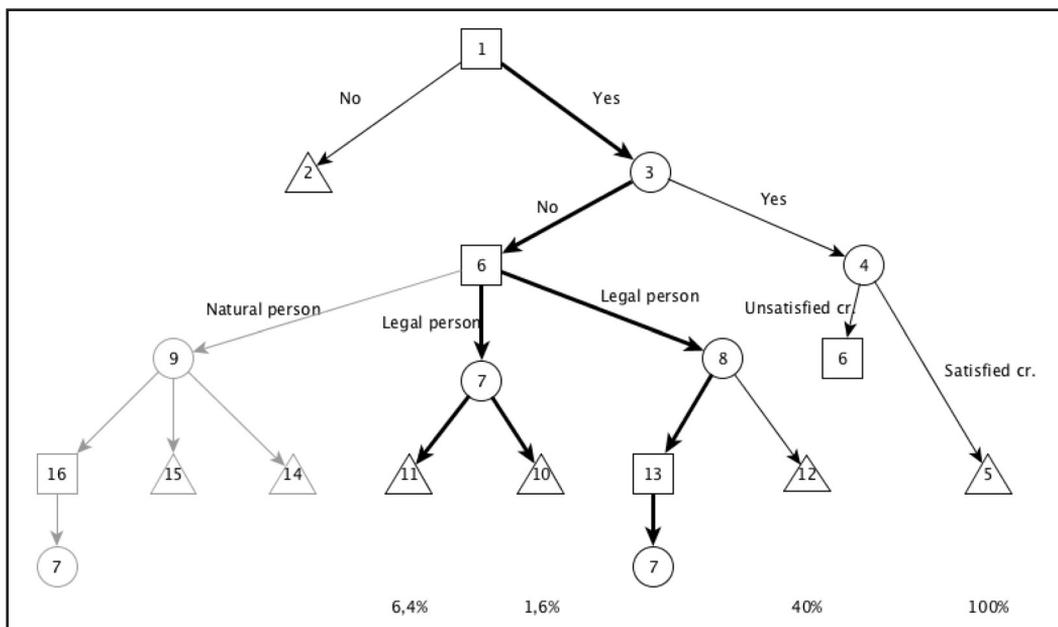


Figure 2. Decision tree of Insolvency proceedings. Source: Authors own work.

claims do not cease to exist, unless stipulated otherwise by the Insolvency Code. Bankruptcy always leads to a liquidation of a debtor that is a legal entity.

By reorganisation, the debtor’s business is preserved and operated pursuant to an approved reorganisation plan under the supervision of the creditors. The creditors’ receivables are paid off gradually.

Debt clearance is only available for debtors who are not entrepreneurs. By debt clearance, all due obligations of the debtor are extinguished subject to the conditions stipulated by the Insolvency Court conducting the proceedings.

The Insolvency Code also provides for special means of dealing the insolvency for special sorts of debtors such as banks and other financial institutions (Baker, McKenzie, 2011).

The decision tree used for the case study, see Figure 2, was made according to legislative division by Insolvency act (The Insolvency Act; 2013).

### 3. Fuzzy reconciliation

Decision-making problem under total ignorance, see Figure 1, gives a system of linear equations  $A \times P = B$ , where the set is a vector of unknown variables, e.g. probabilities, splitting ratios, penalties, etc., is the identity matrix of decision tree and is a vector of numerical constants based on the balance equations, see e.g. (Doubravský, Dohnal, 2015).

Let us imagine a situation where decision makers have isolated information, e.g. probabilities,

Table 1. Importance of nodes.

Level	Node	Importance of nodes	Level	Node	Importance of nodes
I	1	Proposal to bankruptcy	V	9	Debt clearance
VI	2	Rejection of the proposal	VI	10	The cost of the assets
II	3	Bankrupt	VI	11	Creditors
III	4	Moratorium	VI	12	Fulfilling the reorganization plan
VI	5	Meet the demands of creditors	VI	13	Failure reorganization plan
IV	6	Insolvency	VI	14	Sale of debtors assets
V	7	Liquidation	VI	15	Paying off to creditor
V	8	Reorganization	VI	16	Unsatisfied creditors

Source: Authors own work.

splitting ratios, penalties, etc.; the decision-making problem is solved under *partial ignorance* (topology of decision tree and isolated information are known), see e.g. (Doubrovský, Dohnal, 2015). An incomplete set of isolated specific probabilities  $R \equiv (R_1, R_2, \dots, R_h)$  has  $h$  elements. Each element of the set  $R$  can be expressed by fuzzy set, verbal value, etc. and can be formally interpreted as an equation.

The concept of the total ignorance represented by metaheuristics helps to incorporate a set of isolated specific information items within a general framework of metaheuristics; see e.g. (Doubrovský, Dohnal, 2015).

Decisions are often based on heuristics, see e.g. (Mousavi, Gigerenzer, 2014), (Sundar, Singh, 2008). This paper is based on an assumption that decision makers are ready to accept some general heuristics based on common sense reasoning. There are many possible heuristics, which can be mutually contradictory.

This paper is based on the heuristics (1) which reflect some features of common sense reasoning.

*A longer decision tree sub-path is less probable.* (1)

The algorithm studied in this paper is based on a strong analogy between a water flow through a one root tree system of pipes and the decision tree of the same topology. Therefore a methodology is needed to quantify the missing set of information. This problem can be described by general metaheuristics in (Meluzín, Doubrovský, Dohnal, 2012). Let us suppose that one litre of water is pumped into the root node of the decision tree each second and there is no accumulation of water inside the tree. The consequence is that one litre of water must leave the tree through its terminal nodes each second; see e.g. (Poláček, 2015). Flows through all branches of the tree under study must be balanced. The relevant balance equation for a node with *one* flow in, *k* flows out is written:

$$IN_i = \sum_{j=1}^k OUT_j, \quad (2)$$

where  $IN$  is flow into  $i$ th node and  $OUT$  are flows out of  $j$ th node.

There is a relationship between a flowrate through node and a probability of this node. Then a reinterpretation of the heuristic is:

*The flowrate of water through a node is equal to its probability.* (3)

The following classical axiomatic definition of probability can be found in a number of standard texts (Dimitri, Tsitsiklis, 2008). Let  $A$  be an event, and  $\Omega$  is a sample space. A probability  $p(A)$  of event  $A$ , must satisfy three axioms:

$$\text{for all } A \in \Omega, p(A) \geq 0. \quad (4)$$

$$p(\Omega) = 1. \quad (5)$$

$$p(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^n p(A_i). \quad (6)$$

The relevant water flows through a node satisfies the axioms (4)–(6).

This paper is based on the following definition of the splitting ratio  $\alpha_{i,j}$  from  $i$ th node to  $j$ th node, see (Doubrovský, Dohnal, 2015).

Variable  $p_j$  of  $j$ th terminal for  $j \in N$  is a flowrate of water through  $j$ th node where  $N$  is set of all nodes. The value  $P_r$  of a root node always equals one.

$$P_r = 1. \quad (7)$$

Non-root node probability is

$$P_j = P_i \cdot \alpha_{i,j}, j \in (N - T), \quad (8)$$

where  $i$  represents nearest upstream node (the sub-root node of the sub-tree) and  $T$  Set of terminals. The equation (8) is based on the balance equation (2).

The set of  $N - T$  linear equations (8) (the set of balance equations) where the set  $P$  is a vector of unknown variables and the splitting ratios  $\alpha$  are numerical constants can be easily solved.

The answer to the question how to incorporate additional information into total ignorance gives reconciliation.

The problems of reconciliation are very important and have been studied for more than 30 years, see e.g. (Watson, 1994). Reconciliation is a solution of an over-specified set of linear equations (8), see (Doubrovský, Dohnal, 2015):

$$A \times P = B \cup P = R, \quad (9)$$

where  $P = R$  is the set of additional probabilities. The set of equations (6) has  $n + h$  equations and  $n$  variables  $P$ . The set of equations (9) has nearly always no solution. An objective function  $Q$ :

$$Q = \sum_{j=1}^h d_j, \quad (10)$$

is chosen which is minimized. This function (10) is usually a sum of deviations  $d$  (11):

$$d_j = P_j - R_j, \quad (11)$$

where  $j = 1, \dots, h$ . A methods of linear programming is used to solve the following problems (12):

$$\begin{aligned} \min Q \\ \text{s.t. } a_1 P_1 &= b_1 \\ &\vdots \\ a_n P_n &= b_n \\ P_{n+1} &= R_{n+1} \\ &\vdots \\ P_{n+h} &= R_{n+h} \end{aligned} \quad (12)$$

The reconciliation can be solved by a well-known algorithms of linear programming; see *e.g.* (Fedrizzi *et al.*, 1991; Li, Wan, 2013; Tan *et al.*, 2007; Tapia, Tan, 2014).

For example, only the probability of edge 2→6 (splitting ratio)  $p_{2 \rightarrow 6}$ , see Figure 1, is known and is expressed by a fuzzy set which is characterized by triangular grades of membership. The triangular grades of membership can be expressed by a triplet  $(a, b, d)$ , where  $a < b < d$ , see (Doubravský, Dohnal, 2015). Then the set of additional probabilities contains only one element  $R \equiv (R_i)$  and the additional probability R1 can be written as follows:

$$R_1 = (0.26, 0.29, 0.45). \quad (13)$$

These probabilities were chose based on evaluation of decision tree see Figure 1.

In some of the nodes of the decision tree, we are unable to determine the exact probabilities for the decision. Therefore, before entering into the calculation of an average values, we have determined, after consultation with an expert, probability intervals for each such item see *e.g.* (13).

The equation (13) can be transformed into four linear inequalities, see (Doubravský, Dohnal, 2015) by introducing two vectors of slack variables.

The set of over-specified linear equation (9) with the objective function (10) where the set  $A \times P = B$  is obtained by the heuristic H1 and  $P = R$  is the set of four linear inequalities of the additional information (13) is solved as follows:

$$\begin{aligned} p_{2 \rightarrow 4} &= 0.26, \\ p_{2 \rightarrow 5} &= 0.29, \\ p_{2 \rightarrow 6} &= 0.45. \end{aligned} \quad (14)$$

The decision tree, see Figure 1, can be evaluated the probabilities (14).

$$\begin{aligned} LNV_1 &= 0.26 \times 1,000,000 + 0.29 \times 300,000 + \\ &\quad + 0.45 \times (-600,000) = 70,700, \\ LNV_2 &= 0, \\ DNV_1 &= \max [LNV_1, LNV_2] = [70,700; 0] = 70,700. \end{aligned} \quad (15)$$

The decision maker chooses the lottery no. 2, it means he/she chooses the variant – *Make the investment*.

### 4. Case study

MSV Metal Studénka as a manufacturer of die forgings for over 110 years, the largest commodity of manufacturing are railway wagons. MSV Metal Studénka Inc. (hereinafter only “debtor”) came in 2011 into insolvency proceedings because of unpaid debts amounting to 74,817,812 CZK. Due to the fulfilment of the conditions set by law (The Insolvency Act; 2013) for reorganization – more than 50 employees and an annual turnover of over 50 mil CZK, the creditors were able to agree on a solution through insolvency reorganization of the company.

#### Solving by reconciliation

In the following section is described the case of the decision tree, where are known only some probabilities. The following table shows the profits of the individual edges/variant of the decision and the relevant entered probabilities, same as mentioned above see (13). The probabilities on selected nodes were chosen after consultation with insolvency administrator to cover further range of realistic cases.

Numeric conversion is seen in the Table 2, 3.

The Table 4 shows the profits of the individual variants of the decision and the relevant known probabilities. Which are calculated by basic algorithms for determination of the evaluation of a decision tree where the probabilities at each successive node are multiplied by the total possible benefit.

Table 2. Conversion of probabilities to fuzzy.

	a	b	c
A <sub>3,6</sub>	0.88	0.94	0.99
A <sub>3,4</sub>	0.01	0.05	0.1
A <sub>8,12</sub>	0.92	0.93	0.98
A <sub>8,13</sub>	0.02	0.061	0.08

Source: Own work.

Table 3. Splitting ratio.

Branch	Splitting ratio	Branch	Splitting ratio
1–2	0,974	7–10	0,5
1–3	0,026	7–11	0,5
3–4	0,05	8–12	0,939
3–6	0,95	8–13	0,061
4–5	0,944	9–14	0,416
4–6	0,056	9–15	0,417
6–7	0,406	9–16	0,167
6–8	0,313	13–7	1
6–9	0,281	16–7	1

Source: Own work.

Table 4. Profit and the specified probabilities.

Branch	Probability of Branch	Probability of refund	Profit (mil. CZK)	Branch	Probability of Branch	Probability of refund	Profit (mil. CZK)
1-3	1.00			6-8	0.31	0.40	8.81
1-2	0.00			6-9	0.28		
3-6	0.95			7-10	0.50		1.17
3-4	0.05			7-11	0.50		1.17
4-5	0.94	1.00	70.3	8-12	0.94		8.28
4-6	0.06	0.25	1.12	8-13	0.06		0.53
6-7	0.41	0.08	2.33	9-16			

Source: Own work.

Table 5. Average of the profit.

Branch	3-4	6-7	6-8	1-3	1-2
Average Profit (mil. CZK)	71.32	2.33	8.81	27.49	0

Source: Own work.

Table 6. Final comparison.

Branch	3-4 Moratorium	6-7 Bankruptcy	6-8 Reorganization	1-3 Insolvency	1-2 Individual	Average Profit (mil. CZK)
Reconciliation	71.32	2.33	8.81	27.49	0	
Average rec. refund %	95.3%	3.1%	11.8%	36.73%	0.00%	
Average refund % from known prb. & water prb.;	35.94%	4.30%	5.91%	15.37%	0.00%	
Average % refund	65.62%	3.70%	8.86%	26.05%	0.00%	

Source: Own work.

In Table 5 can be seen calculated profits for selected routes from a decision tree, where route 1-3 is the average result of previous sub results for all possible ways in insolvency proceedings. The route 1-2 is 0 because of unfulfilled of legal conditions for individual debt clearance.

## 5. Conclusion

Now it can be clearly seen the comparison the results from previous study with already known probabilities and calculation with water probabilities (Poláček, 2015) to a new results calculated with method of reconciliation. The best solution for creditor is that a debtor will decide to use the way of moratorium and pay back almost the 66% of its claims, but in the real life si-

tuation is the reorganisation the only way that can satisfy the debtor and creditor. Creditor will get something about 9% of its claims and debtor can carry on with his business. This scenario is only possible with fulfilment of every law conditions for reorganisation (Table 6).

A closer look at the final table, it may seem that the fair refunding from the debtor entity is minimal, which is not far from the real situation. The redistribution of amounts owed to creditors is crucial, whether it is a secured creditor or unsecured. Secured creditors, *i.e.* such a creditor whose claim is secured debtor for some thing or the right of the debtor's assets are satisfy 100% of the liquidation proceeds, after deducting amounts attributable to reward manager and management costs, and monetization. It could mean that for unsecured creditors may be even smaller total refund.

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